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CONVECTIVE HEAT AND MASS TRANSFER OF REACTING PARTICLES AT LOW PECLET NUMBERS

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The problem of convective diffusion to a spherical particle in a gas is solved under the condition that the surface chemical reaction rate depends on the reagent concentration near the surface.

The first five terms of the asymptotic expansion in the low Peclet number have been obtained for the medium Sherwood number. Certain specific cases of the surface reaction have been analyzed.

At low Peclet numbers the problem of heat and mass transfer of a solid sphere around which a stationary Stokes stream flows, was first investigated by the method of joined asymptotic expansions in [1]. Constancy of the concentration was assumed far from the particle and on its surface. For the medium Sherwood number the first five terms of the asymptotic expansion were obtained. The extension of this problem to the case of a particle of arbitrary shape was made in [2], where a three-term expansion in the Peclet number was obtained for the medium Sherwood number. An analogous problem was considered in [3, 4] for a sphere, where expressions [5] obtained by the method of joined asymptotic expansions in the low Peclet number were utilized for the fluid velocity field, Convective diffusion to a sphere and particle of arbitrary shape around which a homogeneous translational stream flows during the progress of an isothermal reaction of the first kind on its surface was examined in [6, 7]. The mass transfer of a sphere during the progress of a chemical reaction of the first and second orders on its surface is investigated in [8]. The problem with arbitrary surface reaction kinetics was considered in [9] in the case of Stokes flow around the sphere.

It is assumed that the Reynolds number $R = aU/\nu$ and Peclet number $P = aU/D$ are small (for a gas the Schmidt number is $Sc = \nu/D = O(1)$. A chemical reaction with a finite reaction rate $F(c^*)$ proceeds on the particle surface where the function F is governed by a heterogeneous reaction mechanism. Thus, for a reaction of order $\varkappa$ $F = ka^{-1}c_\infty D(c^*/c_\infty)^\varkappa$.

The process of reagent transport is determined by the convective diffusion equation and the boundary conditions which have the following form in dimensionless variables in a spherical r, θ coordinate system coupled to the particle:

$$\frac{P}{r^2}\frac{\partial(\psi,\ c)}{\partial(r,\ \mu)} = \Delta c,\quad c = \frac{c_\infty - c^*}{c_\infty},\quad \mu = \cos\theta, \tag{1}$$

$$r\to\infty,\quad c\to 0;\quad r = 1,\quad \partial c/\partial r = f(c),\quad f(c) \equiv -a(c_\infty D)^{-1}F(c^*). \tag{2}$$

Here a and U are selected as the scales of the dimensionless quantities.

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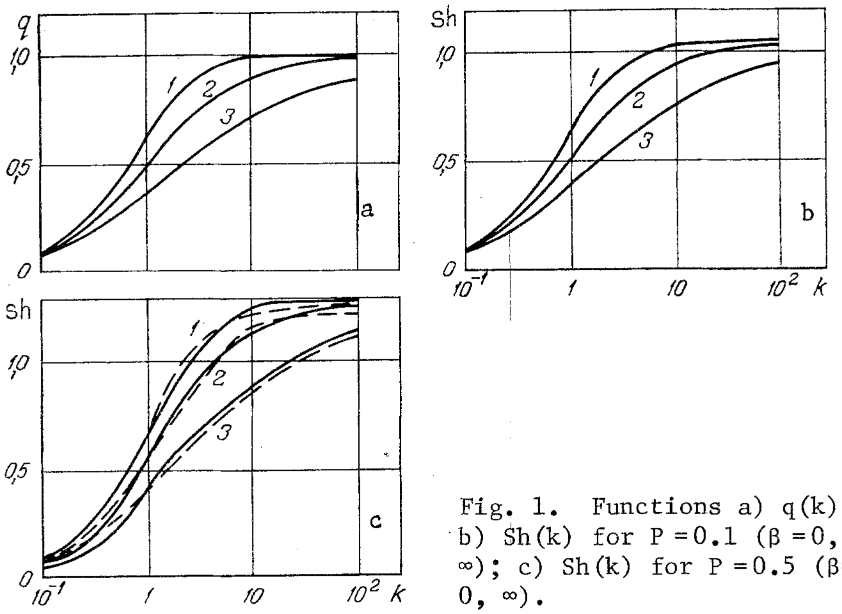

Fig. 1. Functions a) q(k); b) Sh(k) for P = 0.1 (β = 0, ∞); c) Sh(k) for P = 0.5 (β = 0, ∞).

Expressions obtained for drops by the method of joined asymptotic expansions [10] were used for the velocity field

$$\psi = \psi_0 + R\psi_1, \quad \psi_0 = \frac{1}{2}(r-1)\left[r - \frac{1}{2}\frac{\beta}{\beta+1}\left(1+\frac{1}{r}\right)\right]\sin^2\theta,$$

$$\psi_1 = \frac{1}{8}\frac{3\beta+2}{\beta+1}\psi_0 - \frac{1}{16}\frac{3\beta+2}{\beta+1}(r-1)\left[r - \frac{1}{2}\frac{\beta}{\beta+1} - \frac{1}{10}\frac{\beta}{(\beta+1)^2}\left(\frac{1}{r}+\frac{5\beta+6}{r^2}\right)\right]\sin^2\theta\cos\theta. \tag{3}$$

The value $\beta = \infty$ corresponds to a solid sphere, and $\beta = 0$ to a gas bubble. The expression (3) is valid in the inner flow domain $r \leq O(R^{-1})$. Moreover, the problem (1)-(3) is supplemented by a formula for the stream function in the inner stream domain for $r \geq O(R^{-1})$ (the appropriate expression is presented in [10]).

The boundary value problem (1)-(3) was investigated by the method of joined asymptotic expansions in a low Peclet number. Here the whole flow domain was divided into two subdomains: inner $1 \leq r \leq O(P^{-1})$ and outer $O(P^{-1}) \leq r$. As usual, "compression" of the coordinate $\rho = Pr$ is introduced in the outer domain, and the solution is sought separately in each subdomain in the form of inner and outer expansions. The boundary condition on the particle surface was used in constructing the asymptotic solution in the inner domain, and the boundary condition at infinity in constructing the solution in the outer domain; the unknown constants that occur during the solution are determined by joining.

Obtaining and solving the equations governing the terms of the outer and inner asymptotic expansions is described in detail in [1-9]. Omitting intermediate computations, we present only the final results for the main characteristics of the mass transfer process of drops (or solid particles) with a gas-medium Sherwood number stream:

$$\mathrm{Sh} = \frac{I}{4\pi a D c_\infty} = -\frac{1}{2}\int_{-1}^{1}\left(\frac{\partial c}{\partial r}\right)_{r=1} d\mu = q + \frac{q}{2}\frac{\lambda}{\lambda+1}P$$

$$+ \frac{q}{6}\frac{3\beta+2}{\beta+1}\frac{\lambda}{\lambda+1}P^2\ln P + q\left\{\frac{1}{24}\frac{27\beta^2+37\beta+12+12\lambda\beta^2+15\lambda\beta+4\lambda}{(\beta+1)^2(2+\lambda)}\right.$$

$$- \frac{1}{1+\lambda}\{[717\lambda^2\beta^2 + 3099\lambda\beta^2 + 3240\beta^2 + 4440\beta + 1440 + 1120\lambda^2\beta$$

$$\left. + 480\lambda^2 + 4520\lambda\beta + 1200\lambda]\,[2880(\beta+1)^2(2+\lambda)]^{-1} + A\}\right\}P^2$$

$$+ \frac{1}{12}q\frac{3\beta+2}{(\beta+1)(1+\lambda)^2}\left(\lambda^2 - q\frac{\lambda_1}{1+\lambda}\right)P^3\ln P + O(P^3),$$

$$A=\frac{\lambda}{12}\left\{\frac{3\beta+2}{\beta+1}\left[(\mathrm{Sc}+1)^2(\mathrm{Sc}-2)\ln(1+\mathrm{Sc}^{-1})-\mathrm{Sc}^2+\frac{1}{2}\mathrm{Sc}+\frac{25}{6}-2\gamma\right]-\frac{3\lambda}{1+\lambda}\right\}+\frac{1}{8}\lambda_1 q\left[\frac{1}{(1+\lambda)^2}+\frac{1}{48}\frac{(3\beta+4)^2}{(\beta+1)^2(\lambda+2)^2}\right],$$

$$\lambda=[f'(x)]_{x=q},\quad \lambda_1=[f''(x)]_{x=q}. \tag{4}$$

Here I is the total diffusion flow per particle; $\gamma=0.5772...$, Euler constant; and q, root of the algebraic equation

$$-q=f(q). \tag{5}$$

The expressions obtained extend the results of [1-9]. In particular, for $\beta=\infty$ and arbitrary f the three first terms of the expansion for the number Sh were obtained in [9], which agree for $f=-k(1-c)$ with the results in [6-8].

Dependences of the Sherwood number (Sc = 1) on the reaction rate constant k in the case of progress of the reaction on the particle surface according to a power law $f(c)=-k(1-c)^{\varkappa}$ are represented in the figure for values of $\varkappa=1/2$, 1, 2 (curves 1, 2, and 3, respectively) and $\beta=0$, ∞ (the dashed lines in Fig. 1c correspond to the value $\beta=0$, and the solid lines to $\beta=\infty$; all the curves in Figs. 1a and b correspond to $\beta=0$, ∞). Here the dependence of the root q(k) is presented in Fig. 1a. The value $\varkappa=1/2$ corresponds to a heterogeneous reaction of carbon with oxygen proceeding on the surface of a coal particle [11], while for $\beta=0$ the corresponding curves characterize the heat transfer of bubbles in liquid metals where the Schmidt number is $\mathrm{Sc}\approx 1$.

NOTATION

a, particle radius; U, velocity of particle motion; ν, viscosity; D, diffusion coefficient; c^*, concentration; c_∞, concentration far from the particle; k, reaction rate constant; $\varkappa$, order of the reaction; β, ratio between the viscosities of the drop and the surrounding liquid; ψ, dimensionless stream function; Sh, Sherwood number; $R=aU/\nu$, Reynolds number; $P=aU/D$, Peclet number, and $\mathrm{Sc}=\nu/D$, Schmidt number.

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